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Loukas ( ) Conceleiere and Salas ،(۲۰۰۳)Gonzalez

-    ( ) Salas et al.   ( ) and Vasiliades

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and ( ) Shiau and Shen   .

-    ( ) Salas et al.   ( ) Gonzalez Valdez

Mishra and   ( ) Hong Wu and Wilhite

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Sklar,) (Copula)

mohsen_m@modares.ac.ir :   *

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σ_n μ_n

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(Vasiliades, 2004; Shiau, 2006   :

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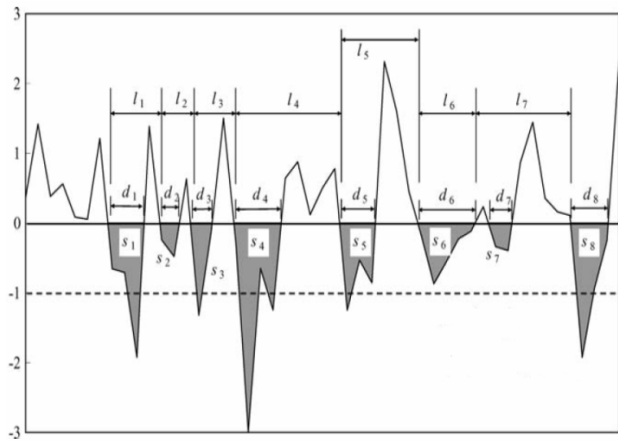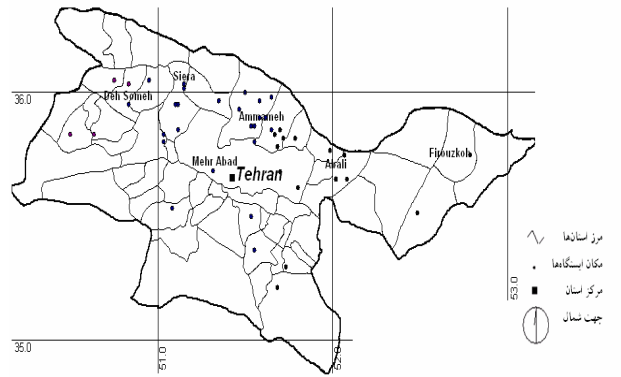
$$S = -\sum_{i=1}^{D} SPI_i \qquad ( )$$

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d_i s_i .

i l_i i

. $(i+1)$ (Sheskin, 2000)

Pilon et al.,)

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(**Shiau, 2006**)

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$$SPI_n = \frac{[P_0 + \sum(P_{-i}) - \mu_n]}{\sigma_n} \qquad ( )$$

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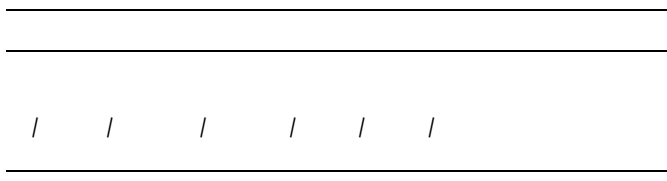
P_{-i} P_0

$$f_{X,Y}(x,y) = c(F_X(y), F_Y(y)) f_X(x) f_Y(y) \quad (\ )$$

c

$$c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v}, 0 < u < 1, 0 < v < 1 \qquad (\ )$$

$[0,1]$

v   u

H ( ) Sklar .

C

( ) Joe . G_Y F_X

( )Nelsen

G_Y F_X

y   x

C

Joint and

$\overline{R} = R \cup \{-\infty, \infty\}$

(Marginal Distributions)

$$H(x,y) = C(F_X(x), G_Y(y)) \qquad (\ )$$

G_Y F_X .

C

R_G R_F $R_F \times R_G$

$f_Y(y)$ $f_X(x)$. G_Y F_X

$$\rho_s = 12 \int_0^1 \int_0^1 [C(u,v) - uv] du\, dv$$

Y   X

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$$\frac{[1+(\theta-1)(u+v)] - \sqrt{[1+(\theta-1)(u+v)]^2 - 4uv\theta(\theta-1)}}{2(\theta-1)}$$

$$1 - [(1-u)^\theta + (1-v)^\theta - ((1-u)(1-v))^\theta]^{\frac{1}{\theta}}$$

$$(u^{-\theta} + v^{-\theta} - 1)^{\frac{-1}{\theta}}$$

$$\frac{-1}{\theta} \ln[1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1}]$$

$$uv \exp\{[(-\ln u)^{-\theta} + (-\ln v)^{-\theta}]^{\frac{-1}{\theta}}\}$$

$$uv \exp(-\theta \ln u \ln v)$$

$$\exp\{-[(-\ln u)^\theta + (-\ln v)^\theta]^{\frac{1}{\theta}}\}$$

$$\frac{uv}{1 - \theta(1-u)(1-v)}$$

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$\lambda e^{-\lambda d}, d \geq 0$

$$\frac{s^{\alpha-1}}{\beta^\alpha \Gamma(\alpha)} e^{\frac{-s}{\beta}}, s \geq 0$$

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$$\ln L(d,s;\lambda,\alpha,\beta,\theta) = \ln L_c(F_D(d),F_S(s);\theta) + \qquad (\ )$$
$$\ln L_D(d;\lambda) + \ln L_S(s;\alpha,\beta)$$

$F_D(d)$ $\qquad F_S(s)$

$\ln L_c(.)$.

( ) $\qquad \lambda \quad \beta \quad \alpha$

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Contour) $\qquad S \quad D$

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$$P(D \geqslant d, S \geqslant s) = 1 \quad F_D(d) \quad F_S(s) + C(F_D(d),F_S(s)) \quad (\ )$$

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$$P(S \le s \mid D \ge d) = \frac{F_S(s) - C(F_D(d),F_S(s))}{1 - F_D(d)} \qquad (\ )$$

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$$P(D \le d \mid S \ge s) = \frac{F_D(d) - C(F_D(d),F_S(s))}{1 - F_S(s)} \qquad (\ )$$

$E(L)$

$X \leq x_t$

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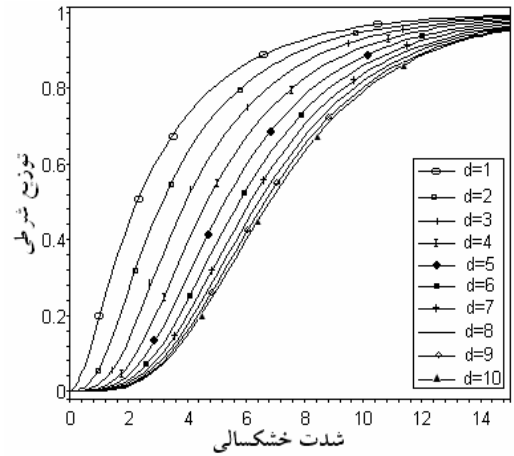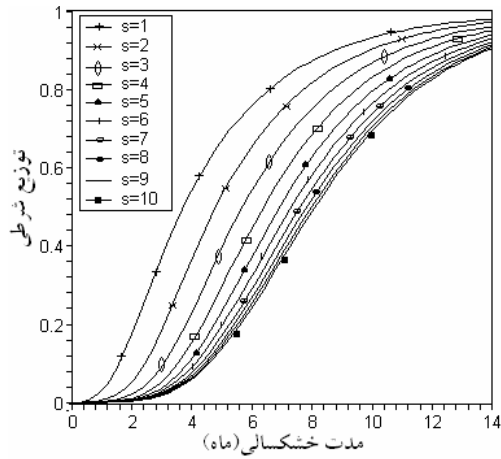
( ) Shiau and Shen

$$T_D = \frac{E(L)}{1 - F_S(s)} \quad , \quad T_S = \frac{E(L)}{1 - F_D(d)} \quad (\ )$$

$E(L)$

/	/	/	/	/	/	$E(L)$

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$$T_{DS} = \frac{E(L)}{P(D \geq d, S \geq s)} = \frac{E(L)}{1 - C(F_D(d), F_S(s))} \quad (\ ) \quad\quad (\ ) \quad (SPI$$

$$T'_{DS} = \frac{E(L)}{P(D \geq d \text{ or } S \geq s)} = \frac{E(L)}{1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s))} \quad\quad (\quad)$$

$d = 3$

( ) Shiau .

$s = 3$

$T'_{DS} = /$ $T_{DS} = /$ $P(D \geq d, S \geq s)$ $P(D \geq d, S \geq s)$

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$$T_{D|S \geq s} = \frac{T_s}{P(D \geq d, S \geq s)} = \frac{E(L)}{[1 - F_S(s)][1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s))]}$$
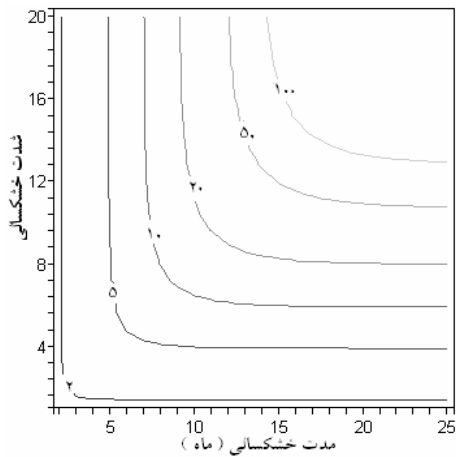
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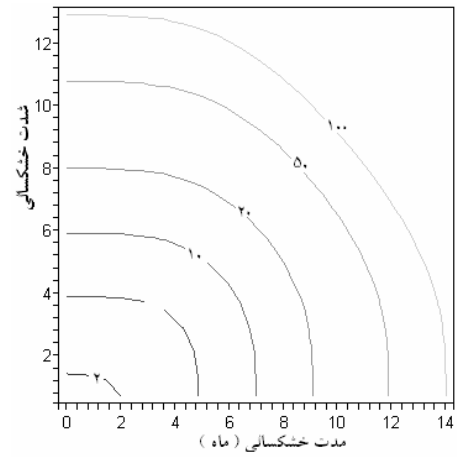$$T_{S|D \geq d} = \frac{T_s}{P(D \geq d, S \geq s)} =$$

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$$\frac{E(L)}{[1 - F_D(d)][1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s))]}$$

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T'_{DS} �‍

T_{DS} ٍ
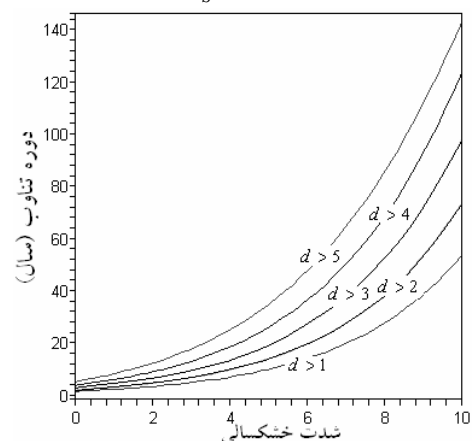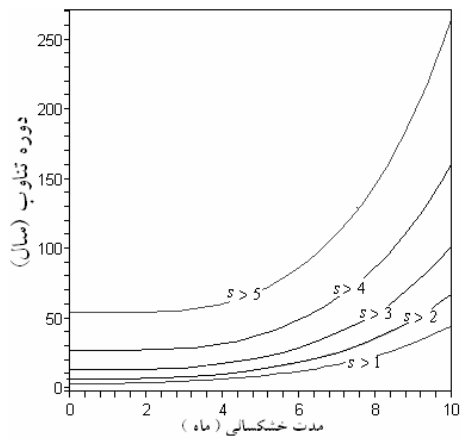
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